Rendering of Surface and Volume Details in Volume Data*

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Abstract:
Aiming at the detail rendering in volume data, a new volume illumination model, called Composed Scattering Model (CSM), is presented. In order to enhance different details in volume data, scattering intensity is decomposed into volume scattering intensity and surface scattering intensity with different weight functions. According to the Gauss probability distribution of gray and gradient of data, we propose an accurate method to detect the materials in a voxel, called composed segmentation. In addition, we discuss the principle of constructing these weight functions based on the operators defined in composed segmentation. CSM can generate images containing more details than most popular volume rendering models. This model has been applied to the direct volume rendering of 3D data sets obtained by CT and MRI. The resultant images show not only rich details but also clear boundary surfaces. CSM is demonstrated as an accurate volume rendering model suited for detail enhancement in volume data sets.

Keywords: volume rendering, volume illumination model, ray casting algorithm

1. Introduction

Currently, displaying of 3D data sets, especially 3D scalar data sets including CT and MRI data, is the main research direction in scientific visualization. The methods include contour extraction or reconstruction [1, 2, 3], voxel-based rendering under surface illumination model [4, 5], and direct volume rendering under volume illumination model [6, 7, 8, 9, 14]. Due to its high fidelity, direct volume rendering attracts more research attentions and becomes the most highlighting research direction in scientific visualization.

In direct volume rendering, volume illumination models are distinguished into analogue based one and equation based one. Analogue based volume illumination model is constructed according to physics supposition. Such as, the varying density emitters model [8] supposed that density field consisted of lighting particles and density was the number of particles per volume unit. The classification and mixture model [7] supposed that each voxel was composed of one or several materials and different material attributes, such as light intensity and transparency, were mixed by their percentages. These models are straightforward and easy to be implemented. But the results under these models are

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limitative and lack of details. For example, the classification and mixture model assumes that there are
no more than two material intensity distributions (e.g. X-ray absorptions of only two neighbouring
different tissues considered in CT) overlapped. In CT, intensity distribution ranges from air, fat, soft
tissue to bone. Drebin's model can only determine the percentages of two successively distributed
materials, such as the mixture of air and fat, or the mixture of fat and soft tissue, or the mixture of soft
tissue and bone. It can not determine the percentages of two separately distributed materials, such as fat
and bone. On the other hand, the equation based model computes visual parameters, transparency and
light intensity, according to the equations in optical physics.

The early works on equation based model are the rendering of random medium, especially on gas
objects, such as cloud, fog, mist, vapor, etc. Blinn[10], Kajiya[6] and Max[11] studied the attenuation
of ray crossing through the random medium. Their researches resulted in a series of scattering phase
functions in different materials and rendering algorithms aiming at blur boundaries as cloud and fog.
Jaffey [15] firstly adopted this model in 3-dimensional volume rendering, called source attenuation
model. Based on the transfer equation [13], Krueger [14] formally proposed the accurate equation based
volume rendering model. He employed the Monte Carlo random simulation method to compute the
intensity of ray in the density field. His method is accurate but very time-consuming. He introduced a
general way to map data features in data field. But in order to enhance different details in data sets, we
need different mapping functions. Unfortunately, these functions are very difficult to be designed. From
the point of view of sampling to reserve details in volume data, Sakas [12] proposed volume sampling
based on pyramid mip-maps structure. In [16], Kaufman selected a 4D hypercone filter to calculate the
sampling value according to the filter's value in object space, and presented the corresponding alias free
ray tracing algorithm. But this method is object dependent and it is impossible to precalculate the filter
of all primitives. It is evident that volume sampling costs much more time than point sampling.

In this paper, we put forward a new volume illumination model, called Composed Scattering Model
(CSM), also based on the transfer equation but aiming at detail approximation. In CSM, details are
classified into volume detail and surface detail. Rendering should show the exact strength and position
of these details. In order to display these two kinds of detail in volume, we propose a method, that
scattering intensity is decomposed into volume intensity and surface intensity with different weight
functions, instead of Monte Carlo random simulation rendering and volume sampling. According to the
Gauss probability distribution of gray and gradient of data, we propose an accurate method to detect the
materials in one voxel, called composed segmentation in this paper. This method is more accurate than
Drebin's model, which only depends on the distribution of gray. Although point sampling is used,
details are preserved. As a result, CSM not only renders the realistic boundary surfaces and volume in
data sets, but also maintains most of details within them, i.e. aiming at the detail enhancement in direct
volume rendering.

Ray casting algorithm is suitable for CSM. We discuss the ray casting implementation of CSM. CSM
has been adopted to display a lot of volume data sets of CT and MRI and the results are encouraging.

2. Composed Scattering Model (CSM)

CSM is derived from transfer equation [13] under some assumptions. The transfer equation in transport
theory is applied to calculate the gain and loss of ray intensity while the ray crosses through a
continuous random medium. This equation can be used to interpret many physical phenomena. In
addition, it is used to calculate the intensity of light ray or microwave in atmosphere and biological
medium.
From the point of view of obtaining clear details in volume and boundary surfaces, it is reasonable to suppose that the field we considered in this paper is a low albedo field, and not a high albedo field such as cloud or fog with blur boundary. The specific simplifying conditions are:

1. single scattering at all sampling points,
2. directly illuminated by the light source,
3. sharing same physical properties in one voxel.

As Figure 1 shows, in voxel $i$ in the medium field, $I(r_i, \vec{e}_z)$ is the radiative intensity at point $r_i$ along direction $\vec{e}_z$. According to the transfer equation [13],

$$I(r_i, \vec{e}_z) = I_i(r_i, \vec{e}_z) + I_d(r_i, \vec{e}_z)$$  \hspace{1cm} (1)

where $I_i(r_i, \vec{e}_z)$ is collimated intensity, which is the intensity of incident ray $R$ at $r_i$ along direction $\vec{e}_z$; $I_d(r_i, \vec{e}_z)$ is diffuse intensity, which is the scattering intensity of all particles in volume at point $r_i$ along direction $\vec{e}_z$.

The contribution of $I(r_{i-1}, \vec{e}_z)$ reaching at $r_i$ makes up of the collimated radiative intensity $I_i(r_i, \vec{e}_z)$ at point $r_i$. According to [13],

$$I_i(r_i, \vec{e}_z) = I(r_{i-1}, \vec{e}_z) \cdot e^{-\int_{r_{i-1}}^{r_i} \sigma(r) \cdot ds} = I(r_{i-1}, \vec{e}_z) \cdot t_i$$  \hspace{1cm} (2)

$$t_i = e^{-\int_{r_{i-1}}^{r_i} \sigma(r) \cdot ds} = e^{-\sigma_i(r_i) \cdot l_i}$$  \hspace{1cm} (3)

where $l_i$ is the transparency of interval $l_i$ from $r_{i-1}$ to $r_i$, $\sigma_i(r_i)$ is the extinction coefficient at the sample point within interval $l_i$.

Diffuse intensity can be decomposed into two parts [13]:

$$I_d(r_i, \vec{e}_z) = I_s(r_i, \vec{e}_z) + I_q(r_i, \vec{e}_z)$$  \hspace{1cm} (4)

where $I_s(r_i, \vec{e}_z)$ is the radiative contributions along direction $\vec{e}_z$ scattered by particles in interval $l_i$ from $r_{i-1}$ to $r_i$ directly illuminated by light sources, called scattering intensity; $I_q(r_i, \vec{e}_z)$ is the radiative contributions along direction $\vec{e}_z$ emitted by particles in interval $l_i$ from $r_{i-1}$ to $r_i$, called source intensity.

Each voxel in data field may consist of one material, or several materials. If a voxel only contains one material, no boundary surface exists in this voxel. Otherwise, boundary surface exists. In these two different situations, scattering intensities are quite different. In CSM, we introduced two kinds of scattering to distinguish them, volume scattering and surface scattering. Scattering intensity are decomposed into volume scattering intensity and surface scattering intensity. The weighted summation of these two kinds of intensity composed the total scattering intensity in a voxel, described as Equation (5).
\[ I_s(r_s, \vec{e}_s) = W_s^i \cdot I_s^i(r_s, \vec{e}_s) + W_s^v \cdot I_s^v(r_s, \vec{e}_s) \]  
(5)

where \( I_s^i \) and \( I_s^v \) are surface scattering intensity and volume scattering intensity, \( W_s^i \) and \( W_s^v \) are the weight functions of \( I_s^i \) and \( I_s^v \) respectively.

This decomposition form is reasonable in optical physics. According to the forward scattering theory, scattering intensity at one point is equivalent to the linear composition of a background scattering and a forward scattering. In 3 dimensions, the distribution of scattering intensity forms a general ellipsoid with two maximum intensity and two minimum intensity directions. We use Equation (5) to approach this kind of distribution. Under the simplifying conditions in this paper, only single scattering is considered. Therefore, the decomposition of Equation (5) is correct at least in form.

In optical physics, the scattering intensity at point \( r_s \) along direction \( \vec{e}_s \) (seeing Figure (1)) is represented as Equation (6).

\[ I_s(r_s, \vec{e}_s) = \sigma_s(r_s) \cdot p(\vec{e}_s, \vec{e}_s') \cdot I_0 \]  
(6)

where \( \sigma_s(r_s) \) is the scattering term and \( p(\vec{e}_s, \vec{e}_s') \) is the phase function.

Then, the volume scatter intensity at point \( r_s \) is,

\[ I_s^v = I_0 \cdot \sigma_s^v(r_s) \cdot p_s^v(\vec{e}_s, \vec{e}_s') = I_0 \cdot F(\rho(r_s)) \]  
(7)

Since no boundary surface exists, volume scattering is a kind of isotropy scattering, i.e.

\[ p_s^v(\vec{e}_s, \vec{e}_s') = 1, \quad \sigma_s^v(r_s) = F(\rho(r_s)) \]  
(8)

where \( \rho(r_s) \) is the density value at point \( r_s \), \( F(x) \) is an encoding function which maps density to radiative intensity. In section 5, we discuss a specific form of \( F \) function, which indicates the isotropy scattering radiative details at a sample point under different illumination situation.

On the other hand, surface scattering is a kind of anisotropy scattering. Therefore, we represent the surface scattering intensity \( I_s^i \) as equation (9).

\[ I_s^i(r_s, \vec{e}_s) = I_0 \cdot \sigma_s^i \cdot p_s^i(\vec{e}_s, \vec{e}_s') \]  
(9)

where phase functions \( p_s^i(\vec{e}_s, \vec{e}_s') \) is shown as Equation (10) to enhance the details on boundary surface.

\[ p_s^i(\vec{e}_s, \vec{e}_s') = \left| \left( \vec{N} \cdot \vec{e}_s' \right) \right| + \left( \vec{N} \cdot \frac{\vec{e}_s' + \vec{e}_s}{|\vec{e}_s' + \vec{e}_s|} \right)^n \]  
(10)

where \( \vec{N} \) is the gradient vector at point \( r_s \), i.e. the normal vector of boundary surface at point \( r_s \).

Considering the front and back surfaces of boundary surfaces, we use the absolute value of \( (\vec{N} \cdot \vec{e}_s') \) in equation (10).

Many image boundary detectors can be chosen as weight functions, such as gradient operator, Laplacian operator, Marr-Hildreth operator, etc. Among them, the gradient operator is the simplest one.
Let the gradient at \( \mathbf{r}_s \) is \( (\mathbf{g}_x, \mathbf{g}_y, \mathbf{g}_z) \), the gradient boundary operator is defined as
\[
\mathbf{g} = \sqrt{\mathbf{g}_x^2 + \mathbf{g}_y^2 + \mathbf{g}_z^2}.
\]
It is evident that \( W^g_i \) and \( W^g_s \) are the functions of \( p \) and \( \mathbf{g} \). In section 4, based on the Gauss probability distribution of \( p \) and \( \mathbf{g} \), \( W^g_i \) and \( W^g_s \) are represented as the operators in composed segmentation.

The source intensity plays a role like the ambient intensity in local illumination model, which is the radiative intensity emitted by particles from \( \mathbf{r}_{i-1} \) to \( \mathbf{r}_i \).
\[
I_q(r_s) = F_q(\rho(r_s)) = \rho \cdot I_0
\]
(11)
Thus, the equations from (1) to (11) make up of our Composed Scattering Model (CSM) in this paper.

3. Ray Casting Rendering

CSM is very suitable for ray casting rendering algorithm. As Figure 2 shows, for each pixel in screen, a ray casting from the view point passes through the data field. Along each ray, the intensity is accumulated at all sample points.

According to CSM, the intensity at point \( \mathbf{r}_i \) is
\[
I(r_i, \mathbf{e}_s) = I(r_{i-1}, \mathbf{e}_s) \cdot I_i + I_{d}(r_i, \mathbf{e}_s)
\]
(12)
where,
\[
I_{d}(r_i, \mathbf{e}_s) = \int_0^{t_i} d[[I_0 \cdot \sigma_i(r_i) \cdot p(\mathbf{e}_s, \mathbf{e}_s') + I_q(r_i, \mathbf{e}_s)] \cdot e^{-\sigma_i ds}]
\]
\[
= \int_0^{t_i} [I_0 \cdot \sigma_i(r_i) \cdot p(\mathbf{e}_s, \mathbf{e}_s') + I_q(r_i, \mathbf{e}_s)] \cdot e^{-\sigma_i ds}
\]
\[
= I_0 \cdot \sigma_i(r_i) \cdot p(\mathbf{e}_s, \mathbf{e}_s') + I_q(r_i, \mathbf{e}_s) - \frac{\sigma_i}{\sigma_i}
\]
\[
= W^s_i \cdot I^s_i(r_i, \mathbf{e}_s) + W^v_i \cdot I^v_i(r_i, \mathbf{e}_s) + I_q(r_i, \mathbf{e}_s) \cdot (1 - t_i)
\]
(13)
Thus, the total intensity reaching to the view point is:
Therefore, the summation order of intensity composition is from front to back along viewing direction.

When accumulated transparency is less than a predefined threshold \(\epsilon\), i.e. \(\prod_{j=i}^{n} t_j < \epsilon\), summation process may be stopped. Therefore, this summation order is very convenient for implementation by ray casting algorithm in direct volume rendering.

For the transparency \(t_i = e^{-\sigma_i / h}\) of interval \(I_i\) in equation (3), where \(\sigma_i = \sigma_s + \sigma_a\), it is obviously that \(\sigma_s\) and \(\sigma_a\) are the scattering and absorption coefficients of collimation incident ray. In order to simplify the computation and enhance the boundary effect, we suppose that \(\sigma_i\) at point \(r_s\) is dependent on the density value and the gradient value at this point.

Let \(\sigma_i = \sigma_i, \rho, g\) \((\rho + gp)\), then,

\[
\sigma_i, \rho, g = \frac{\sigma_s + \sigma_a}{\rho + gp} = \sigma_{i, \rho, g} + \sigma_{a, \rho, g}
\]

where \(\sigma_{i, \rho, g}\) and \(\sigma_{a, \rho, g}\) are constants. Thus, we can directly calculate the transparency \(t_i\) of collimation incident ray according to the value of \(\rho(r_s)\) and \(g(r_s)\) at point \(r_s\).

### 4. Composed Segmentation

In order to clearly display the different objects in data sets, segmentation is essential. In CSM, we propose a segmentation method related to the weighted intensity calculation in Equation (5).

To each material, its gray level relates to a probability distribution, e.g. Gauss distribution. The probability indicates the reliability of which current sampling value belongs to one material. In CSM, we suppose that the probability belonging to material \(O_i\) is a 2-dimension Gauss distribution of gray and gradient, as shown in Equation (16).

\[
P(X, O_i) = \frac{1}{2\pi \sqrt{|C|}} e^{-\frac{1}{2}(X-M_i)^T C^{-1} (X-M_i)}
\]

where \(X\) is a vector of (gray, gradient), \(M\) is the peak position of \(X\), \(C\) is the covariance matrix of gray and gradient. For example, if \(O_i\) is soft tissue, the range of gray is [90, 120], then we set \(M_i = (105, 10)\). Thus, the covariance matrix of soft tissue is Equation (17).
If a field consists of four kinds of materials, there are four typical probability distribution cases to one voxel, shown in Chart 1. In each case, the order of probability ranges from high to low. Case 1 is a typical case of only containing one material. In this case, the probability of one material is much higher than the other. It is evident that this voxel belongs to this material. Case 2 and Case 3 are the cases in which a voxel is composed of two or more than two materials. In the above three cases, a great probability drop can be found between two adjacent materials which is much greater than the other drops.

Chart 1: Four Typical Cases

Equations (18) to (20) define two relative operators $\text{In}(X)$ and $\text{Boundary}(X)$, which determine the ownership attributes of materials in a voxel. $\text{Drop}(X)$ function returns the greatest probability drop, where $\text{diff}$ is the difference of two adjacent material probabilities. Diff should be larger than a threshold to prevent from regarding case 4 as a case which contains boundary surface. In fact, Case 4 is an exception, in which the probabilities are approximated to each other. Therefore, not only this voxel does not belong to a material but also a boundary surface does not exist.

$$\text{In}(X) = \{i | P(X, O_i) = \max\{P(X, O_k)\}\}$$

(18)

$$\text{Drop}(X) = \{P(X, O_i) | \text{diff} = \max\{P(X, O_i) - P(X, O_{i+1})\} \& \text{diff} > \text{threshold}\}$$

(19)

$$\text{Boundary}(X) = \{< O_i > | P(X, O_i) >= \text{Drop}(X)\}$$

(20)

According to the above operators, $W_s^V$ and $W_s^B$ in Equation (5) can be represented as Equation (21) and (22).

$$W_s^V(X, O_i)_{i(\text{In}(X))} = P(X, O_i) \cdot \prod_{j=0}^{j=i} (1 - P(X, O_j))$$

(21)
\[ W^s_t(X, <O_l>)|_{\text{Boundary}(X)} = \prod_{O_j \in <O_l>} (1 - P(X, O_j)) \]  

**Gradient**

Figure 3 Iso-contour of Probability

Figure 3 is the iso-contour of probability of all materials in CT data set, which shows the principle of \( W^s_t(X, O_i) \) and \( W^v_t(X, <O_l>) \). In this figure, the iso-contour set of each material is a 2-dimensional Guass probability distribution. Points A, B, C indicate three different situations. Point A is closed to the peak of the reliability of FAT, \( P(A, \text{Fat}) \) is great enough to set \( ln(A) \) to FAT. Thus, \( W^v_t \) is much greater than \( W^s_t \). Volume scattering intensity is much greater than surface scattering intensity, i.e. volume details is enhanced. Point B is a typical boundary surface point. At this point, \( \text{Boundary}(B) \) is much greater than \( <\text{SOFT TISSUE, BONE}> \), \( W^v_t \) is much greater than \( W^s_t \). As a result, surface scattering intensity is much greater than volume scattering intensity, i.e. surface details is enhanced. At point C, its gray value falls in the range of soft tissue, but with a great gradient. As we know, if the gradient is not great, the point must be in soft tissue. Great gradient indicates that this point belongs to boundary surface. According to the return value of \( \text{Boundary}(X), <\text{FAT, BONE}> \), we detect that the point locates at the boundary surface of fat and bone. Thus \( W^v_t \) is high. Comparing with Drebin's model, this model can segment the mixture of separately distributed materials, as point C indicates. Therefore, more details are accurately enhanced than Drebin's models.

With this composed method, segmentation and rendering becomes two coherent processes. Furthermore, different weights reflect the scales of different details in the volume, i.e. enhancing surface detail or enhancing volume detail.

### 5. Implementation and Results Discussion

Based on CSM, we implemented a ray casting algorithm for direct volume rendering. The images in this paper are generated by this algorithm using the following data sets:

- CT data set, 128 x 128 x 197 x 8 bit (sloth)
- MRI data set, 150 x 200 x 192 x 8 bit (head)
The sampling steps are evenly spaced along ray. The step distance is selected so that each intersected voxel should be sampled once at least. The sampling value and its gradient are calculated by using three dimensions linear interpolation, where the difference is center difference.

In CSM, the \( F \) function in equation (6) and (7) is still undecided. In fact, \( F \) is an encoding function to data set. We use the HLS encoding [15] to convert the density of volume into object color, that is

\[
F(\rho, I_0, g, \phi),
\]

where \( \rho \) is the sampling value at current point, \( I_0 \) is the light source intensity at current point, \( g \) is the gradient, \( \phi \) is a coefficient. Density defines hue, the scale of the gradient defines saturation, and light is the intensity of the light source. This encoding method enhances the effect of changes in density. In order to avoid zero gradient, coefficient \( \phi \) is included.

The rendering results are shown in Figure (4) and (5).

Figure (4)* uses the sloth chest CT data set. These images show the chest bone and the organs in the sloth chest. In image (a), rib protrudes on the skin, which is the effect of segmentation of fat and bone boundary surface. Due to the fact that the breastbone belongs to cartilage, it is very difficult to display its precise structure with the general threshold method for CT bone segmentation, as shown in image (b). CSM generates image (c) which clearly displays the continuous boundary surface of the breastbone including its position and structure. In image (d), we display the organs in the chest although we did not know their biological terminology.

Figure (5)* shows the images of the head MRI data set. The thickness and the strength of the boundary surface ensure the displaying of semi-transparent head skin with subcutaneous blood vessel (seeing image (a)). If there is no thickness, the subcutaneous blood vessel can not be included. On the other hand, if there is no strength, the subcutaneous blood vessel can not be displayed so clearly although it is included. With composed segmentation, we clearly display the inner structure of brain, including cerebrum, cerebellum, eyeball and medulla oblongata as shown in image (b). In image (c) and (d), we display the results of cutting which also display the cross section of head and brain. In the cross section of the head in image (c), we can clearly see the details on the cross section, such as gum. In image (d), we can see the inner of the brain by cutting the brain into two parts. With the undergoing CSM, the cross section in image (c) and (d) clearly show the different layers of the brain and head skin.

6. Conclusion

From our point of view of detail rendering, we proposed CSM for direct volume rendering. In order to approximate two different details in data set, i.e. volume detail and surface detail, scattering intensity is decomposed into surface scattering intensity and volume scattering intensity. The intensity of current point is the weighted composition of these two scattering intensity. We discussed the composed material probability distribution based on gray and gradient. Also, we derived the weight functions from the operators in composed segmentation. The boundary surface of separately distributed materials is exactly located. Therefore, CSM approaches more details than commonly used models. The illumination intensity and segmentation are regarded as two coherent processes in volume rendering. The results of CSM demonstrated that this model realized a realistic volume rendering.

* See page C-510 for Figures 4 and 5.
References

[10] J. Blinn, "Light Reflection Functions for Simulation of Clouds and Dusty Surfaces", SIGGRAPH'82