Learning Individual-Specific Dictionaries with Fused Multiple Features for Face Recognition

Shu Kong, Student Member, IEEE, Donghui Wang, Member, IEEE.

Abstract—Recent researches emphasize more on exploring multiple features to improve classification performance. One popular scheme is to extend the sparse representation-based classification framework with various regularizations. These methods sparsely encode the query image over the training set under different constraints, and achieve very encouraging performances in various applications, especially in face recognition (FR). However, they merely make an issue on how to collaboratively encode the query, but ignore the latent relationships among the multiple features that can further improve the classification accuracy. It is reasonable to anticipate that the low-level features of facial images, such as edges and smoothed/low-frequency image, can be fused into a more compact and more discriminative representation through some relationships for better FR performances. Focusing on this, we propose a unified framework for FR to take advantage of this latent relationship and to fully make use of the fused features. Our method can realize the following tasks: (1) learning a specific dictionary for each individual that captures the most distinctive features; (2) learning a common pattern pool that provides the less-discriminative and shared patterns for all individuals, such as illuminations and poses; (3) simultaneously learning a fusion matrix to merge the features into a more discriminative and more compact representation. We perform a series of experiments on public available databases to evaluate our method, and the experimental results demonstrate the effectiveness of our proposed approach.

I. INTRODUCTION

With the recent endeavor made by computer vision researchers, an increasing number of feature types have been designed to describe various aspects of visual characteristics. In practice, multiple visual data for one individual can also be derived through heterogenous sensors, e.g. visible light cameras, inferred cameras and laser range finders. Facing these multiple information cues, in particular, researchers begin to jointly study these multiple information sources for object recognition tasks, and very good classification performances are reported [5]. Even though it is widely believed that classification performance can benefit from jointly studying multiple features, effective processing and analysis methods are still badly in need.

On another track, researchers employ the dictionary learning (DL) model for classification tasks under the supervised fashion [10], [13]. By exploring the label information of the objects, DL-based methods, which are a particular sparse coding model, learn the classification-oriented dictionary mainly in two ways [12]: either (1) directly making the dictionary discriminative, such as learning class-specific sub-dictionary for each class, or (2) making the sparse coefficients discriminative to propagate the discrimination power to the dictionary. Even if these DL-based classification methods achieve very promising or even state-of-the-art performances on many public datasets for various applications, especially for face recognition (FR), they only work on a single feature type, e.g. the original gray-level facial image, rather than multiple informative features. Thus, they cannot easily exploit multiple features of the same physical object and their possible semantic correlations for better performance.

To address the aforementioned problems, i.e. how to take advantage of the multi-feature information and how to employ the powerful sparse coding framework for classification, researchers have proposed several methods [26], [27], [24]. Yuan and Yan propose a multi-task joint sparse representation based classification method (MTJSRC), which, with the help of multiple features, treats the recognition task as a multi-task problem and each feature type is one task [26]. They assume the sparse coefficients of all the features share the same sparse pattern, and identify the query image according to the reconstruction error accumulated over all the feature types. However, the assumption is too strict and is not held in practice. Therefore, Zhang et al. propose a joint dynamic sparse representation classification method (JDSRC) [27] to address this problem. They argue that the same sparsity pattern is shared among the coefficients at class-level, but not necessarily at atom-level. Yang et al. also address this problem by proposing a relaxed collaborative representation method (RCR), which assumes the sparse codes with respect to all the feature types should be similar in appearance [24]. All the three methods elaborately consider the sparsity pattern among the coefficients of different feature types, and achieve very encouraging performances on FR. However, these methods merely use the training data as a pre-defined dictionary, which can be very large when the number of training data increases and thus makes the recognition computationally expensive. Moreover, for classification, directly dealing with all the available features can be too redundant for classification, thus, the recognition stage will be very time-consuming. In the classification respect, these multi-feature cues have some semantic connections with each other, and if we estimate their semantic relationship, we can learn a

This work is supported by 973 Program (Project No.2010CB327904) and Natural Science Foundations of China (No.61071218).

Shu Kong and Donghui Wang are with the Department of Computer Science and Technology, Zhejiang University, Hangzhou 310027, China {aimerykong, dhwang}@zju.edu.cn

1Strictly speaking, the joint dynamic sparse coding method is dealing with object recognition through multiple observations. But their framework can be extended to recognition on multi-modal features as said by the authors in [27].
more compact and more discriminative dictionary for better classification performance.

Moreover, there are a multitude of low-level features that can have the potential to enhance the recognition accuracy, such as edges of the face, smoothed image and the low-frequency of the face image. But no methods have been proposed to explore these. Concentrating on the above issues, in this paper, we propose a unified framework to explore the relationship among these features for better FR performance. Our method realizes the following goals:

1) automatically fusing the features into a more compact and more discriminative representation of the facial images;
2) learning a compact and discriminative individual-specific dictionary for each person to capture the most distinctive features;
3) learning a common pattern pool which provides the shared and essential patterns for reconstruction of the face images of all persons.

The rest of this paper is organized as follows. In Section II, we briefly review several approaches that motivate ours. The proposed method is elaborated in Section III, followed by experimental evaluation in Section IV. Finally, we conclude our paper in Section V with discussions.

II. RELATED WORK

Sparse representation-based classification (SRC) [21] is a far-reaching method under sparse coding theory. It achieves very encouraging performances on face recognition with robustness to illumination changes and occlusions. Suppose there are C classes of individual faces, let \( D = [X_1, \ldots, X_N] \in \mathbb{R}^{p \times N} \) be the set of original training set, where \( X_c \in \mathbb{R}^{p \times N_c} \) consists of all the \( N_c \) column-vector-represented training samples from class \( c \) and \( N = \sum_{c=1}^{C} N_c \). SRC treats \( D \) as a pre-defined dictionary. Given the query facial image \( x \in \mathbb{R}^p \), it identifies \( x \) through a two-stage procedure: (1) sparsely encode \( x \) over \( D \) via an \( \ell_1 \)-norm minimization \( a = \text{argmin}_a \|x - Da\|_2^2 + \lambda \|a\|_1 \), where \( \lambda \) is a scalar to balance the reconstruction error and the sparse degree; (2) identifying \( x \) to class \( c \) such that \( c = \text{argmin}_c \|x - X_c \delta_i(a)\|_2^2 \), where \( \delta_i(\cdot) \) is a vector indicator function that extract the elements corresponding to the \( i \)-th training set \( X_i \).

However, as SRC directly uses the original training set as the dictionary, this pre-defined dictionary will incorporate too much redundancy as well as noisy and trivial information that can degenerate the performance. Additionally, when the training data grows in number, the computation of sparse coding will become a major bottleneck. To circumvent this problem, Yang et al. propose a Metaface learning method to learn a class-specific dictionary \( D_c = [d_{c1}, \ldots, d_{cK_c}] \in \mathbb{R}^{p \times K_c} \) for each individual [23]:

\[
D_c = \text{argmin}_{D_c, A_c} \left\{ \sum_{c=1}^{C} \|X_c - D_c A_c\|_F^2 + \lambda \|A_c\|_1, \right. \\
\left. \text{s.t. } \|d_{cj}\|_2 = 1, \forall j = 1, \ldots, K_c, \right. \\
\right.
\]

where \( \|A_c\|_1 \) is defined as the summation of \( \ell_1 \)-norm of all the columns of \( A_c = [a_{c1}, \ldots, a_{cN_c}] \in \mathbb{R}^{K_c \times N_c} \), i.e. \( \|A_c\|_1 = \sum_{c=1}^{C} \|a_{cj}\|_1 \). This method concatenates all the sub-dictionaries as an overall dictionary \( D = [D_1, \ldots, D_C] \) for classification, the same as the second stage of SRC.

As pointed out in the literature [13], [17], the learned class-specific dictionaries usually share some common patterns, which do not help or even degenerate the discrimination of the facial images but are essential for representation of them. This observation also applies to FR well, as the facial images usually share some common patterns, such as illuminations and poses. Considering this observation, the recent proposed method DL-COPAR by Kong and Wang makes a difference between the class-specific features and the common patterns [13]. It learns \( C \) class-specific dictionaries \( D_c \)'s for each of the \( C \) individuals and a common pattern pool \( D_{C+1} \in \mathbb{R}^{p \times K_{C+1}} \). The two parts constitute the overall dictionary \( D = [D_1, \ldots, D_C, D_{C+1}] \in \mathbb{R}^{p \times K} \) in which \( K = \sum_{c=1}^{C+1} K_c \).

The existing DL-based methods (including DL-COPAR) can be summarized as a general framework (omitting the constraint on dictionary atoms) in dictionary learning stage:

\[
\min_{D, A} \left\{ \sum_{c=1}^{C} \|X_c - D_c A_c\|_F^2 + \lambda \|A_c\|_1 + \eta \|Q(D)\|_F^2, \right. \\
\left. \right. \\
\right.
\]

In Metaface learning, \( C(X, D, A) = \sum_{c=1}^{C} \|X_c - D_c A_c\|_F^2, \phi(A) = \sum_{c=1}^{C} \|A_c\|_1 \) and \( \eta \) is set zero. As for DL-COPAR, the three terms are explained as below:

- \( C(\cdot) \) measures the reconstruction error of the training data by the dictionary. It ensures the dictionary to have the power of representing any face images, and the images from a specific class (say class \( c \) can be well approximated by the collaborative effort of the \( c \)-th sub-dictionary \( D_c \) and the common pattern pool \( D_{C+1} \). Define a selection operator \( R_c = [r_{c1}, \ldots, r_{cK_c}] \in \mathbb{R}^{K_c \times K_{C+1}} \) in which:

\[
r_{cj} = \begin{bmatrix} 0, 0, \ldots, 0, 0, 0, \ldots, 1, 0, \ldots, 0, 0, \ldots, 0 \end{bmatrix}^T, \quad \sum_{m=1}^{j-1} K_m = K_c, \quad \sum_{m=j}^{C+1} K_m = 0 \}
\]

Therefore, we have \( D_c, D_{C+1} = D[R_c, R_{C+1}] \), and \( \sum_{c=1}^{C} \|Q_c\|_F^2 \) selects the coefficient part corresponding to the \( c \)-th sub-dictionary and the common pattern pool, i.e. \( D_c, D_{C+1} \). By denoting \( Q_c = [R_c, R_{C+1}] \), then \( C(\cdot) \) is defined as:

\[
C(X, D, A) = \sum_{c=1}^{C} \left\{ \|X_c - D_c A_c\|_F^2 + \|X_c - D_{C+1} Q_c^T A_c\|_F^2 \right\} \}
\]

- Denote \( Q_c \in \mathbb{R}^{K_c \times K_{C+1}} \), then the term \( \phi(A) \) is defined as:

\[
\phi(A) = \sum_{c=1}^{C} \left\{ \|Q_c^T A_c\|_F^2 + \lambda \|A_c\|_1 \right\} \}
\]

It can be seen that \( \phi(A) \) pushes coefficients \( A \) to be sparse with the \( \ell_1 \)-norm penalty and regularizes \( A \) to
guarantee the irrelevant sub-dictionaries $D_i$’s ($i \neq c$ and $i \neq C + 1$) do not contribute to the representation of the images from class $c$.

- $Q(D)$ works on the dictionary to drive different sub-dictionaries as incoherent as possible:
  \[ Q(D) = \sum_{c=1}^{C+1} \sum_{j=1}^{C+1} \|D_i^c D_j^c\|_F^2. \]

DL-COPAR optimizes the objective function alternatively, and adopts a SRC-like scheme [21] for classification. In our paper, we adopt a similar framework to incorporate and fuse multiple features for FR, as DL-COPAR especially produces very promising performance in FR.

III. OUR METHODOLOGY

In this section, we first introduce how to incorporate multiple features under the dictionary learning framework. Then, we unify the fusion process into the framework. Thirdly, we provide a simple and effective initialization process for the proposed method. Finally, we explain our classification scheme.

A. Exploiting multiple features to learn the dictionary

In recent years, some approaches [24], [26], [27] are proposed to exploit multiple features via some sparse coding techniques to boost classification performance. Beyond doubt, these coding techniques will improve the FR performance to some extent by seriously considering the structural sparse patterns of the coefficients, but they merely use the original training set² as a pre-defined dictionary. As noticed in Section II, this pre-defined dictionary has some drawbacks: (1) incorporating too much redundancy as well as noisy and trivial information, and (2) making the sparse coding process much more time-consuming when the training set becomes large in number. Therefore, learning a compact dictionary for each individual is a preferable alternative.

Suppose we have $K$ types of features³, thus each facial image is represented by a matrix or second-order tensor [11], [20]. Specifically, $x_i^k$ means the $k^{th}$ feature of the $i^{th}$ facial image $X_i$; if this image is from the $c^{th}$ person, then we denote $i \in I_c$. Under the framework of Eq. 2, we can easily extend the first term of Eq. 3 of DL-COPAR to exploit multiple features:

\[ C(X, D, A) = \sum_{c=1}^{C} \sum_{k=1}^{K} \sum_{i \in I_c} \left\{ \| x_i^k - D^k a_i^k \|^2_F + \| x_i^k - D^k Q_i^k a_i^k \|^2_F \right\}. \]  

Similarly, the penalty term over the coefficients and the sub-

³In this paper, we restrict our research under the assumption that all the $K$ features have the same length $p$. Even though the multiple-length features can be transformed to be the same dimensional by some techniques, e.g. PCA, we do not exploit these as the tensor representation of the same-length features can preserve the spatial correspondence information [22].

²Here the so-called training set does not merely mean the original facial images any more, but the generated features of the images.

dictionaries can be easily extended respectively:

\[ \phi^*(A) = \sum_{c=1}^{C} \sum_{k=1}^{K} \left\{ \|Q_i^k a_i^k\|^2_F + \lambda \|A_i^k\|_1 \right\}, \]

\[ Q(D) = \sum_{c=1}^{C} \sum_{j=1}^{C+1} \sum_{k=1}^{K} \|D_i^c D_j^c\|_F^2. \]

By replacing the above defined terms into Eq. 2, we derive a unified formulation to incorporate multiple features to learn multiple dictionaries, each one for each feature type. However, these feature types are independent to each other, or the formulation can be cast as running DL-COPAR on each of the $K$ features alone. This means the latent recognition-oriented relationships among the $K$ overall-dictionaries are not explored which can further improve the performance. In the next subsection, we will elaborate how to explore the recognition-oriented relationships.

B. Fusing multiple features

Suppose we have already learned the $K$ dictionaries for each feature type, $D^k \in \mathbb{R}^{p \times D}$ for $k = 1, \ldots, K$, and arrange them to a tensorial representation $D \in \mathbb{R}^{p \times D \times K}$, as illustrated by Fig. 1. It is intuitive to assume each dictionary w.r.t. the specific feature has its own discrimination power and all the feature-specific dictionaries can be connected with each other through some relationships that can generate better classification accuracy. Therefore, our goal is to exploit this relationship among these dictionaries for better FR performance and to lower computational burden. In our work, we assume there is a semantic core dictionary⁴ $B \in \mathbb{R}^{p \times D \times M}$ $(M < K$ or $M \ll K$), which can be linearly extended to all the $K$ dictionaries. In other words, as illustrated by Fig. 2, there is a transformation matrix $W \in \mathbb{R}^{K \times M}$, such that $D = B \times W$. Here $B \times W$ means that the transformation matrix $W$ is applied to the third mode of tensor $B$ (please refer to [11], [20] for details on tensor calculation due to limited space). With the core dictionary $B$ and the transformation $W$, we rewrite Eq. 4 to derive a new term with the across-feature relationship explained by

⁴In this paper, we only consider the semantic meaning across the $K$ dictionaries, and do not over-explore the semantic meaning at atom-level of each dictionary.
\( \mathbb{B} \) and \( \mathbb{W} \):

\[
\mathcal{C}(X, \mathbb{B}, \mathbb{W}, \mathcal{A}) \equiv \sum_{c=1}^{C} \sum_{i \in I_c} \sum_{k=1}^{K} \left\{ \|x^i_k - D^i_k a_k^i\|_F^2 + \|x^i_k - D^i_k Q_k^i a_k^i\|_F^2 \right\},
\]

\( s.t. \mathbb{D} = \mathbb{B} \times \mathbb{W} \),

\( D^k \) is the \( k \)th slice of \( \mathbb{D} \) along the third mode.

From Eq. 6, we can see the dictionaries of these features are connected by the core dictionary \( \mathbb{B} \) under relationship \( \mathbb{W} \). But it should be noted that, if we aim to predict a query image via the \( K \) dictionaries, we still have to deal with all the \( K \) feature types. That is to say \( \mathbb{B} \in \mathbb{R}^{P \times D \times M} \) and \( \mathbb{W} \) bring no computational benefits and they do not necessarily capture the discriminative relationship for better classification. For this reason, we turn to an alternative.

Given a facial image with \( K \) features represented by \( X_i \in \mathbb{R}^{P \times K} \), we let the fusion matrix \( \mathbb{W} \) act on \( X_i \) rather than the \( K \) dictionaries \( D^i \)'s (or \( \mathbb{B} \)), resulting in a compact representation \( Y_i \in \mathbb{R}^{P \times M} \) of this image: \( Y_i = \mathbb{X}_i \times \mathbb{W} \). Put it in another way, the multiple features are fused by the transformation matrix \( \mathbb{W} \). Therefore, we arrive at the final term of our objective function:

\[
\mathcal{C}^*(X, \mathbb{B}, \mathbb{W}, \mathcal{A}) \equiv \sum_{c=1}^{C} \sum_{i \in I_c} \sum_{m=1}^{M} \left\{ \|X_iw_m - B^m a_k^i\|_F^2 + \|X_iw_m - B^m Q_k^i a_k^i\|_F^2 \right\},
\]

where \( w_m \) is the \( m \)th column of \( \mathbb{W} \) in \( \mathbb{W} \in \mathbb{R}^{K \times M} \) and \( B^m \) in \( \mathbb{B} \in \mathbb{R}^{P \times D \times M} \) is the \( m \)th slice of the core dictionary \( \mathbb{B} \in \mathbb{R}^{P \times D \times M} \), corresponding to the \( m \)th fused feature. As \( M < K \) or even \( M < K \), solving sparse coding process in the classification is more computationally efficient and the semantic dictionary \( \mathbb{B} \) can be fully exploited. Accordingly, the regularization on the core dictionary \( \mathbb{B} \) is re-defined as:

\[
\mathcal{Q}^*(\mathbb{B}) \equiv \sum_{c=1}^{C+1} \sum_{j=1}^{C+1} \sum_{k=1}^{K} \|B_{c}^TB_{j}^k\|_F.
\]

\( \psi^*(W) \equiv \text{tr}(W^T(S_w - S_b)W) \).

Intuitively, the above MMC-based term can push the fused features by \( \mathbb{W} \) to be as discriminative as possible. Meanwhile, it can effectively circumvent the small-sample-size problem. But please note that it is not required for \( \mathbb{W} \) to be column-wise orthogonal. Now we arrive at our final objective function by adding this term:

\[
f \equiv \mathcal{C}^*(X, \mathbb{B}, \mathcal{A}) + \lambda \phi^*(\mathcal{A}) + \eta \mathcal{Q}^*(\mathbb{B}) + \omega \psi^*(\mathbb{W}),
\]

where \( \mathcal{C}^*(X, \mathbb{B}, \mathcal{A}) \), \( \phi^*(\mathcal{A}) \) and \( \mathcal{Q}^*(\mathbb{B}) \) are defined in Eq. 7, Eq. 5 and Eq. 8, respectively. \( \lambda \), \( \eta \) and \( \omega \) are constant scalars to balance the contribution of each term. Besides, after the initialization of \( \mathbb{W} \), we have the fused representation of all the facial images, then we use the K-SVD [11] to initialize the \( M \) dictionaries of the core dictionary \( \mathbb{B} \) and the corresponding coefficients \( \mathcal{A} \). This objective function is easy to optimize alternatively over each parameter, and it will converge as updating each parameter is convex when others are fixed.

D. Classification scheme

After the core dictionary \( \mathbb{B} \) and the fusion matrix \( \mathbb{W} \) are learned, we can use them to recognize a query facial image. Recently developed methods, e.g. MTJSRC [26], JDSRC [27] and RCR [24], adopt a sparse representation-based classification (SRC) scheme with different regularizations on the coefficient. In detail, given a query image with \( K \) features \( X \in \mathbb{R}^{P \times K} \), we can first fuse the features with the learned \( \mathbb{W} \in \mathbb{R}^{K \times M} \) into \( Y = \mathbb{X} \mathbb{W} \), and then encode \( Y \) over the
core dictionary $\mathcal{B}$:

$$A = \arg\min_A \sum_{m=1}^{M} ||y^m - B^m a^m||_F^2 + \lambda \phi(A),$$

where $\phi(A)$ is a regularization (different from the DL process) on the coefficients $A = [a^1, \ldots, a^M]$. It is $\phi(A)$ that makes these methods different from each other. Concretely, MTJSRC utilizes an $\ell_2, 1$-norm penalty over the coefficients [26], JDSRC uses group sparsity [27], whereas RCR adopts an overall-similarity term [24]. Note that the order of atoms among the learned dictionaries is crucial for MTJSRC and RCR, as the sparsity pattern relies on the atom order across the dictionaries. Therefore, only the group-sparsity regularization of JDSRC can be directly used in our framework.

In this paper, we also employ a group-level sparse regularizer under the SRC scheme, but using a restricted one by allowing only one class-specific sub-dictionary and the common pattern pool to represent the query image. This constraint enables us to adopt a local sparse coding method for classification [13], i.e. using the individual sub-dictionary and the common pool to reconstruct the facial image:

1) Calculate the reconstruction error over the core dictionary $\mathcal{B}$ w.r.t the $c^{th}$ individual for $c = 1, \ldots, C$:

$$e_c = \min_{A^m} \sum_{m=1}^{M} \{ ||y^m - B^m Q_c^T \alpha^m||_F^2 + \lambda \alpha^m\},$$

2) identify the query image to the person which produces the smallest reconstruction error: $\text{label}(X) = \arg\min_{c} e_c$.

This classification scheme is just a least square problem [28] with a Tikhonov regularization, thus it is much easier and more efficient to implement than solving the ad hoc sparse problems [26], [27], [24].

IV. EXPERIMENT

In this section, we evaluate our method through a series of experiments on three public available datasets:

- Extended YaleB [14] contains 2,414 frontal face images under different illuminations of 38 persons, about 68 images for each individual. The cropped images are resized to $32 \times 32$-pixel resolution for the experiment.
- CMU-PIE [18] is a challenging database due to various illuminations and expressions. In our experiment, we use a subset of this database that contains five near frontal poses (C05, C07, C09 C27, C29) of all the 68 individuals under different illuminations and expressions [3]. Therefore, there are 170 images for each person in total.
- AR database [16] contains two-session data of 70 male and 56 female subjects, each one has 26 pictures with the normalized size as $50 \times 40$-pixel resolution. We use the samples from Session 1 for training and that from Session 2 for testing.

For the first two datasets, we randomly select half of the images (about 32 images per person in Extended YaleB and 85 in CMU-PIE) for training and the rest for testing. We start this section with a discussion on the multiple features and parameters.

A. Features and parameters

As our method fuses multiple features for FR, in this paper, we use $K = 10$ low-level features as illustrated by Fig. 3:

1) The original gray-scale facial image.
2) The corresponding histogram-equalized image.
3) Low-frequency Fourier feature [19] of original image with cut-off frequency 8.
4) Low-frequency Fourier feature of histogram-equalized image with cut-off frequency 8.
5) Sobel edge image [8] of the original image (threshold=15) with $3 \times 3$ mean filter.
6) Sobel edge image of the histogram-equalized image (threshold=15) with $3 \times 3$ mean filter.
7) Canny edge image [4] of the original image (low threshold= 30 and high threshold= 76) with $3 \times 3$ mean filter.
8) Canny edge image of the Low-frequency Fourier feature (as the same threshold above) with $3 \times 3$ mean filter.
9) Canny edge image of the original image (low threshold= 20 and high threshold= 51) with $3 \times 3$ mean filter.
10) Sobel edge image of the original image (threshold=20) with $3 \times 3$ mean filter.

In our experiment, we do not make any differences on the features, but setting the equal weight on them.

In our objective function Eq. 11, we have three parameters to balance the influence of each term in learning the dictionary. For $\lambda$, we set it to make approximate 10% of the coefficients nonzero. $\eta$ decides the incoherence of the individual-specific dictionaries, and we set it to be 1, the same as that in [13]. Larger $\omega$ pushes the fused feature to be more discriminative with each other, therefore, we can set it to a relatively large scalar, say $\omega = 10$ in our experiment.

As for the dictionary $\mathcal{B} \in \mathbb{R}^{p \times D \times K}$, we vary $D$ w.r.t different datasets, as explained by Table I, in which $D_{class}$ and $D_{common}$ denote the number of atoms that constitute each individual-specific dictionary and the common feature pool ($D = #individual \times D_{class} + D_{common}$).

### B. Face recognition

To verify the effectiveness of our method, we compare our method with five closely related and state-of-the-art methods. These methods include holistic SRC (H-SRC) [21] and separate SRC (S-SRC) [27], MTJSRC [26], JDSRC [27] and RCR [24]. H-SRC and S-SRC act as baseline methods that H-SRC concatenates all the features into a single large vector, while S-SRC, as an intuitive extension of SRC, separately encodes each feature and then summarizes the reconstruction error of each cues for FR with the classification scheme of SRC [21]. Note that, MTJSRC only uses two types of features in [26], whereas it performs better on the ten features in this paper than reported in [26]; JDSRC manually selects the face regions in [27] for multi-region FR, yet we run it on the global features to report the results; RCR also considers sophisticatedly segmenting the face images for multi-region FR, but we run RCR for multi-feature FR to report its performance. Moreover, in the above three approaches, there are some parameters (refer to Eq. 2) that control the sparse degree of the coefficients, and we adjust them to get the best performances of each method. All the compared algorithm do not involve any fusion process, but directly encoding the query over the training set (the derived features).

Instead of using all the ten features, we also randomly select $K \leq 10$ features out of the ten for comparison. In detail, we repeat 10 times to randomly select a subset of the features, and each subset is also repeated 5 times, thus 50 times are run for each method on each database. Note that, only our method can fuse the features, and others merely use all the features without any fusion process. We report the averaged accuracy with the standard derivation.

Direct comparisons of FR is listed in Table II. Two settings are included in the table: (1) $K = 5$ features out of the ten are randomly selected and (2) all the $K = 10$ features

<table>
<thead>
<tr>
<th>Method</th>
<th>Extended YaleB $K = 5$</th>
<th>Extended YaleB $K = 10$</th>
<th>CMU-PIE $K = 5$</th>
<th>CMU-PIE $K = 10$</th>
<th>AR $K = 5$</th>
<th>AR $K = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-SRC</td>
<td>97.11 ± 0.32</td>
<td>97.40 ± 0.27</td>
<td>94.39 ± 0.40</td>
<td>95.44 ± 0.34</td>
<td>90.19 ± 0.30</td>
<td>91.46 ± 0.44</td>
</tr>
<tr>
<td>S-SRC</td>
<td>94.20 ± 0.23</td>
<td>94.85 ± 0.34</td>
<td>92.04 ± 0.33</td>
<td>92.61 ± 0.31</td>
<td>87.54 ± 0.63</td>
<td>89.11 ± 0.46</td>
</tr>
<tr>
<td>MTJSRC</td>
<td>98.33 ± 0.40</td>
<td>98.56 ± 0.31</td>
<td>95.61 ± 0.41</td>
<td>96.92 ± 0.24</td>
<td>93.91 ± 0.41</td>
<td>94.61 ± 0.31</td>
</tr>
<tr>
<td>JDSRC</td>
<td>98.50 ± 0.26</td>
<td>99.03 ± 0.27</td>
<td>96.46 ± 0.39</td>
<td>97.51 ± 0.20</td>
<td>94.46 ± 0.39</td>
<td>94.51 ± 0.29</td>
</tr>
<tr>
<td>RCR</td>
<td>98.12 ± 0.30</td>
<td>98.47 ± 0.41</td>
<td>95.83 ± 0.38</td>
<td>97.24 ± 0.30</td>
<td>94.53 ± 0.32</td>
<td>95.24 ± 0.30</td>
</tr>
<tr>
<td>Ours</td>
<td>98.78 ± 0.28</td>
<td>99.63 ± 0.24</td>
<td>97.65 ± 0.41</td>
<td>98.33 ± 0.23</td>
<td>95.76 ± 0.41</td>
<td>96.92 ± 0.43</td>
</tr>
</tbody>
</table>

The weight of each feature can produce different FR performances, as both JDSRC [27] and RCR [24] have discussed the weight. But in our paper, we merely set the weight to be 1 for all the features.
are used for experiment. Note that we set the fused feature number $M = 3$, i.e., our method fuses the $K$ features into $M = 3$ new features for the representation of each facial image. From this table, we can easily see that our method consistently outperforms all the others. It is worth noting that our method fuses the $K$ features into a smaller $M = 3$ new ones, and others just uses all the $K$ features for recognition. When $K = 10$, i.e., all the ten features are adopted for FR, our approach fuses them into a more compact representation with only $M = 3$ new features, and produces more decent FR performances. It demonstrates that the proposed method uses three times smaller feature than the original facial image representation and achieves the highest face recognition accuracy.

To make a more clear comparison, we plot the curve of FR accuracy vs. the number of adopted features in Fig. 4. From these figures, we can see our proposed method always achieves higher FR rate than the others, especially in AR database, ours outperforms the state-of-the-art approaches with a large margin. In appearance, to all the methods, it seems that more features means better FR performance. It is an issue worth studying, and we postpone a brief discussion of it in Section V.

In Fig. 5, we display the three fused features of the five individuals that are previously showed in Fig. 3. The first row illustrates the original facial images, and the following three rows correspond to the three fused features. From this figure, especially from the second and third row, we can see the fused features are robust to illumination changes. Even though there are shadows caused by various illumination angles, these shadows are alleviated in the fused features that provide the resilience to illumination changes. Additionally, from the second row, i.e., the first fused feature, we can see the trivial pixels that bring no discrimination power in the face are smoothed, and the important patterns, e.g., the eyes and the nose, are highlighted. We think it is the pixel-level spatial correspondence of the features represented by tensor attributes to this phenomenon, thus intuitively demonstrating why the fused features can produce more decent FR performances.

C. Face recognition under different fused feature number

In this subsection, we further examine the effects of the fused feature number $M$ on the recognition rate using the three public available datasets. Different from the subsection that the fused feature number $M$ is fixed to be 3, now we vary the number $M$ in the range of $M \in \{1, 2, 3, 4, 5, 6, 7\}$ with all the $K = 10$ features, and the plots in Fig. 6 give the performance of our method on the three datasets. For each of the dataset, we randomly select about 50% facial images per person for training and use the rest for testing. This is different on AR database as used in the previous subsection. All the experiments are repeated 10 times, and the averaged accuracy with standard deviation are plotted as an error bar in Fig. 6.

From this figure, we can see that, when $M$ increases in a range, the FR rates are improved as well in general. But when $M$ reaches some thresholds, the FR performance will saturate. It is easy to explain that larger $M$ means more discriminative fused features are generated. As the regularization term and initialization of the fusion matrix $\mathbf{W}$ is motivated by Fisher criterion [7] and MMC [15], this phenomenon resembles Fisher LDA that when the reduced dimension increases in a reasonable range, the classification becomes more accurate.

V. CONCLUSION

In this paper, we briefly review several recent developed methods that either improve the dictionary learning frame-
work or jointly consider multiple features to achieve better FR performance, and analyze some drawbacks of them. To circumvent these drawbacks, we propose a unified method to fuse various features into a more compact and more discriminative one for better FR. Our approach also learns an individual-specific dictionary for each person and a common feature pool for all the subjects. The common feature pool consists of patterns that are shared by all the individuals, thus making the learned overall dictionary more compact and more discriminative. As for recognition, the proposed method encodes the query (its multiple features) over the dictionary sparsely at group level. But we restrict the coding process to only allow one individual-specific dictionary and the common feature pool to contribute to the approximation. Therefore, we merely use a loose-square fitting criterion for the coding. This coding scheme is effective owing to the help of the common feature pool. Through experimental validation, we demonstrate the proposed method can achieve very decent FR performance with comparison of other closely related and state-of-the-art approaches.

However, compared with MTJSRC, JDSRC and RCR, one limitation of our method is that it only uses global features that are with the same length for fusion. There is no doubt that, despite the global features, the local ones and features that are with the same length for fusion. There is one limitation of our method is that it only uses global related and state-of-the-art approaches. A dictionary learning approach for classifying. This coding scheme is effective owing to the help of the common feature pool to contribute to the approximation.

Therefore, we merely use a loose-square fitting criterion for the coding. This coding scheme is effective owing to the help of the common feature pool. Through experimental validation, we demonstrate the proposed method can achieve very decent FR performance with comparison of other closely related and state-of-the-art approaches.

However, compared with MTJSRC, JDSRC and RCR, one limitation of our method is that it only uses global features that are with the same length for fusion. There is no doubt that, despite the global features, the local ones and features that are with the same length for fusion. There is one limitation of our method is that it only uses global related and state-of-the-art approaches. A dictionary learning approach for classifying. This coding scheme is effective owing to the help of the common feature pool to contribute to the approximation.

Therefore, we merely use a loose-square fitting criterion for the coding. This coding scheme is effective owing to the help of the common feature pool. Through experimental validation, we demonstrate the proposed method can achieve very decent FR performance with comparison of other closely related and state-of-the-art approaches.

However, compared with MTJSRC, JDSRC and RCR, one limitation of our method is that it only uses global features that are with the same length for fusion. There is no doubt that, despite the global features, the local ones and features that are with the same length for fusion. There is one limitation of our method is that it only uses global related and state-of-the-art approaches. A dictionary learning approach for classifying. This coding scheme is effective owing to the help of the common feature pool to contribute to the approximation.

Therefore, we merely use a loose-square fitting criterion for the coding. This coding scheme is effective owing to the help of the common feature pool. Through experimental validation, we demonstrate the proposed method can achieve very decent FR performance with comparison of other closely related and state-of-the-art approaches.

However, compared with MTJSRC, JDSRC and RCR, one limitation of our method is that it only uses global features that are with the same length for fusion. There is no doubt that, despite the global features, the local ones and features that are with the same length for fusion. There is one limitation of our method is that it only uses global related and state-of-the-art approaches. A dictionary learning approach for classifying. This coding scheme is effective owing to the help of the common feature pool to contribute to the approximation.

Therefore, we merely use a loose-square fitting criterion for the coding. This coding scheme is effective owing to the help of the common feature pool. Through experimental validation, we demonstrate the proposed method can achieve very decent FR performance with comparison of other closely related and state-of-the-art approaches.